

# A GRAPH-SEARCH APPROACH ON RESOURCE-CONSTRAINED SCHEDULING PROBLEMS AND ITS APPLICATION TO ADVANCED DRIVER ASSISTANCE SYSTEMS

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Abstract: In this paper we present a problem which is a variation of the resource-constrained project scheduling problem and a graph-based approach to solve it. The problem is defined as resource-constrained scheduling problem (RCSP). Particularly, we apply the approach to the problem of scheduling a large number of driver warnings based on car-to-car communication (also known as cooperative vehicles). Data is presented from the project SIM<sup>TP</sup>, a large-scale field test in the area of the Hessian city of Frankfurt, where 120 cars participate in a number of controlled tests in three main scenarios: the rural road scenario (basic complexity), the motorway scenario (intermediate complexity), and the urban road scenario (high complexity). We argue that, due to its run-time behaviour, our graph-based approach is suitable for the particular application domain at hand. Results are presented in terms of quality of the solution (conflict resolution), runtime behavior and pruning effects to the size of the search tree. In addition to the scenarios derived from the actual field test, a hyper-real stress test is presented to demonstrate the performance of our solution.

## 1 INTRODUCTION

The resource-constrained project scheduling problem (RCPSP) is a combinatorial optimization problem (Blazewicz et al., 1983). Due to its high practical importance, it has been analyzed for several decades now. RCPSP is an extension of the job shop scheduling problem (JSP). (Garey and Johnson, 1979) provides the following definition, which is used as a basis here.

**Definition 1 (JSP).** *The job shop scheduling problem (JSP) consists of a set  $\mathcal{A}$  of activities each with a positive integer-valued duration,  $d_i$ .  $\mathcal{A}$  is partitioned into projects (jobs), and with each project is associated a total ordering on that set of activities. Each activity specifies a resource on which it must execute without interruption. No activities that require the same resource can overlap in their execution.*

Finding a solution to this problem with minimal makespan (difference between minimum start time and maximum end time) is NP-hard. RCPSP is more complicated than JSP, since each task requires not only a processor but also additional scarce resources. Since it can be reduced to the simpler JSP, it is NP-hard as well (Garey and Johnson, 1979; Blazewicz

et al., 1983).

In this paper, we introduce the resource-constrained scheduling problem (RCSP), which is similar but adds additional constraints to start- and endtime of activities. Unlike the original RCPSP, it has a dynamic component from disruption management. (Clausen et al., 2001) defines a disruption as "a state during the execution of the current operation, where the deviation from plan is sufficiently large that the plan has to be changed substantially." In a similar manner, (Kuster et al., 2007) defines disruption management (DM) as "the process of responding to an unforeseen disturbance occurring during the execution of planned and scheduled operations.". According to the author, DM aims at the selection of appropriate repair actions to minimize the negative impact typically associated with disruption.

In the automotive domain, particularly in the field of Advanced Driver Assistance Systems (ADAS), we face the challenge of scheduling a growing number of assistance systems trying to communicate simultaneously with the driver over a limited amount of communication channels. This differs from RCPSP in a way that scheduling priority lies not on the order of activities but on preserving the requested presentation times as good as possible. Moreover, following (Baier

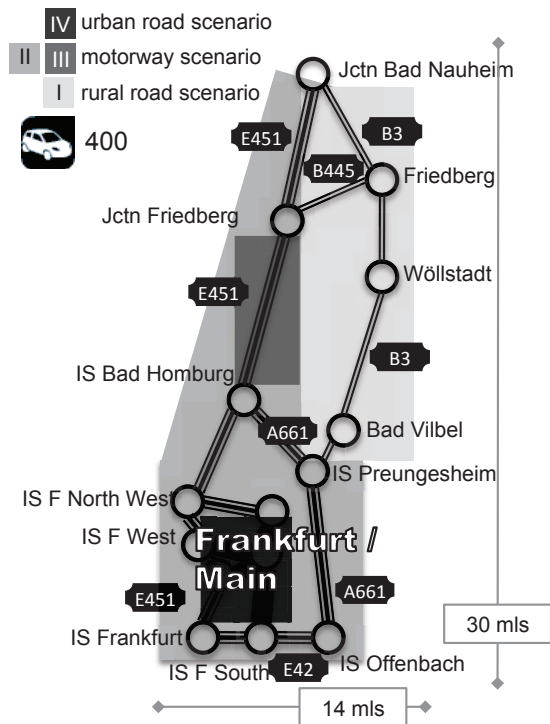


Figure 1: The SIM<sup>TD</sup> test field around Frankfurt/Main, Germany, is comprised of three test scenarios that imply different degrees of resource utilization from low (rural road scenario) to high (urban road scenario). 120 cars are involved in the field test.

et al., 2007), we do not only need to be able to distinguish between plans that satisfy goals and those that do not without providing further means of discrimination between successful plans. For the application at hand, we need to have information about how "good" a plan is, thus "enabling the planner to distinguish between successful plans of differing quality" (ibid.). Furthermore, the nature of the application demands for an anytime behaviour. The desired algorithm to solve the problem should be able to output a solution at any time, which should be as close as possible to the optimal solution.

## 2 BACKGROUND

RCSP consists of a set of tasks (activities) with fixed start times and durations, which should be executed without interruption on a given (limited) resource. Thus, conflicts occur when two or more tasks attempt to use the same resource simultaneously. The objective is to resolve all conflicts while modifying the set of tasks as little as possible.

For RCSP, several algorithms have been pro-

posed, reaching back to the 1960's with a Branch-and-Bound approach by (Johnson, 1967), adopted later by (Stinson et al., 1978; Christofides et al., 1987; Brucker et al., 1998). (Pritsker et al., 1969; Patterson and Roth, 1976) proposed Zero-One programming solutions, while more recent approaches use Linear (Brucker and Knust, 2000), Constraint (Demasse et al., 2005) and Genetic Programming (Kuster et al., 2007).

The Branch-and-Bound approach is closest to our solution presented here. It is based on graph search and effective bounding (e.g. pruning). Linear, Integer and Zero-One-Programming are variations of a standard optimization problem. Genetic programming is a search heuristic which mimics natural selection behaviour.

It is important to keep in mind that our problem presented here differs significantly from the original RCSP and thus, the approaches mentioned here cannot simply be adopted.

For solving RCSP in the given highly dynamic domain, we need an anytime algorithm which finds a good (not necessary perfect) solution very fast. Both tree search and genetic algorithms fulfill these requirements. In this paper, we present a tree search approach with effective pruning.

## 3 APPLICATION DOMAIN

We apply the approach to the problem of scheduling a large number of driver warnings based on car-to-car communication (also known as cooperative vehicles). Particularly, the system is developed for the project SIM<sup>TD</sup>.

The SIM<sup>TD</sup> test field is located in the Frankfurt-Rhine-Main area of Hessen, Germany. With up to 120 vehicles and more than 100 roadside stations installed by the Hessian traffic centre (VZH) and the Integrated Traffic Management Center Frankfurt (IGLZ), car-to-x communication is tested under real conditions. The Frankfurt-Rhine-Main area is an important German traffic hub with major traffic generators such as Frankfurt Airport, Frankfurt Trade Fair and the stadium. Large parts of the area are characterised by high traffic density and therefore allow experiments on all road safety and traffic efficiency functions under normal as well as high load conditions. Figure 1 shows a map of the test field. It is comprised of three scenarios: I Rural road, II / III Motorway, and IV Urban road. The order of the scenarios represents their increasing use of the limited HMI resources. These assumptions are based on the number of applications active in the respective area, the traffic density as well

as the density of road site stations (stationary wireless communication units) used in the field test.

Figure 2 gives an overview on the complexity of the scenarios and specifies the role of RCSP in the given application context. A large number of presentations try to access (potentially) simultaneously a limited resource (the Human-Machine Interface, HMI). Not all requests are equally important. Output channels are distributed over different modalities: multiple visual channels (symbols and main screen), auditory outputs, and text-to-speech (TTS). Hence, unlike traditional presentation planning, we look at a highly dynamic scenario in which presentation requests come in at any time during runtime. As indicated by the dashed triangle, a second resource limitation has to be taken into account: the limited cognitive resource of the driver. Here questions should be addressed like for example, how long the minimal display time for an information should be, or whether or not overlapping presentations on different channels (TTS for one message and simultaneous display of another message) is beneficial. However, this is beyond the scope of this paper. We refer the interested reader to (Cao et al., 2009; Cao et al., 2010; Mahr et al., 2010).

So called *presentation requests* are issued by the "functions" listed on the right side of Figure 2. Functions act as individual subsystems that have no knowledge on the available resources and activities of other functions. Presentation requests are issued ("announced") as soon as the need for a warning becomes apparent. Hence, the scheduler can allocate the HMI resources beforehand (planning time does not delay crucial warning messages). Nevertheless, the scenario involves a multiplicity of presentations and is highly dynamic.

## 4 PROPOSED GRAPH-BASED SOLUTION

In short, our problem is a dynamic set of tasks (presentation requests) simultaneously trying to access a limited resource. Tasks have a start time, duration, a hard minimal duration, a priority, and a desired resource (presentation strategy). We look for a solution which resolves resource conflicts while modifying the tasks as little as possible. A more formal definition is given below.

We propose a two-step graph-based solution to the problem. Step one is transforming dynamic planning to static planning at runtime. Dynamic planning thus is by construction equivalent to having a series of static planning problems. The main problem here is

to identify subsets of the set of presentation tasks and the appropriate order of resolving multiple different conflict sets. Identifying overlapping presentations is a straightforward task at first sight. However, since the solution to that conflict could again cause other conflicts, the appropriate set of tasks to be included in the conflict set is not always obvious. Here we have to solve the trade-off between scope of the solution and runtime, which is heavily depending on the amount of tasks in the conflict set.

Step two of our approach is transforming our problem into a tree search problem for which well known algorithms can be applied.

Let  $\mathcal{T}$  be a set of presentation tasks  $\mathcal{T} = \{t_1, t_2, \dots, t_M\}$ . Let each presentation task  $t_i$  consist of a start time, an end time, a priority and additional information about the presentation such as rendering information, distance to event, etc. which are not of immediate relevance for the scheduling.

**Definition 2** (Scheduling Problem RCSP). *The RCSP  $\mathcal{P}$  is defined as:*

$\mathcal{P} = \langle \mathcal{T}^C, \mathcal{M} \rangle$  where

- $\mathcal{T}^C$  is a set of conflicting presentation tasks,  $\mathcal{T}^C = \{t_1^C, t_2^C, \dots, t_M^C\}$ .
- $\mathcal{M}$  is a set of modifying actions (short: modifications) as conflict resolving strategies,  $\mathcal{M} = \{m_1, \dots, m_N\}$

In our implementation, modifications are postponing, preponing, shortening (beginning or end of task), switching resource (not considered in this paper) and canceling a task.

**Definition 3** (Search Tree). *We define a search tree  $\mathcal{S}(\mathcal{P})$  as follows:*

- The root  $R$  of the tree contains the set of conflicting tasks  $\mathcal{T}^C$
- Each edge  $e$  is a pair of a presentation task and a modification applied to it,  $e = \langle t_i^C, m_j \rangle$
- A cost function  $p(E)$ , which defines a positive integer penalty for each edge, based on the severity of the modification (see Definition 5).
- Each node is the result of the modification of the previous edge to the parent node
- The penalty  $p(n)$  of a node  $n$  is the sum of the penalties of all edges on the branch leading to it
- A node  $n$  containing a presentation task set  $\mathcal{T}$  without conflicts is a leaf.

By design, the leaf with the lowest cost is the best solution to our problem. Using the tree  $\mathcal{S}(\mathcal{P})$  described in Definition 3, we can now search the solution with the Breadth First Search (BFS) Algorithm 1.

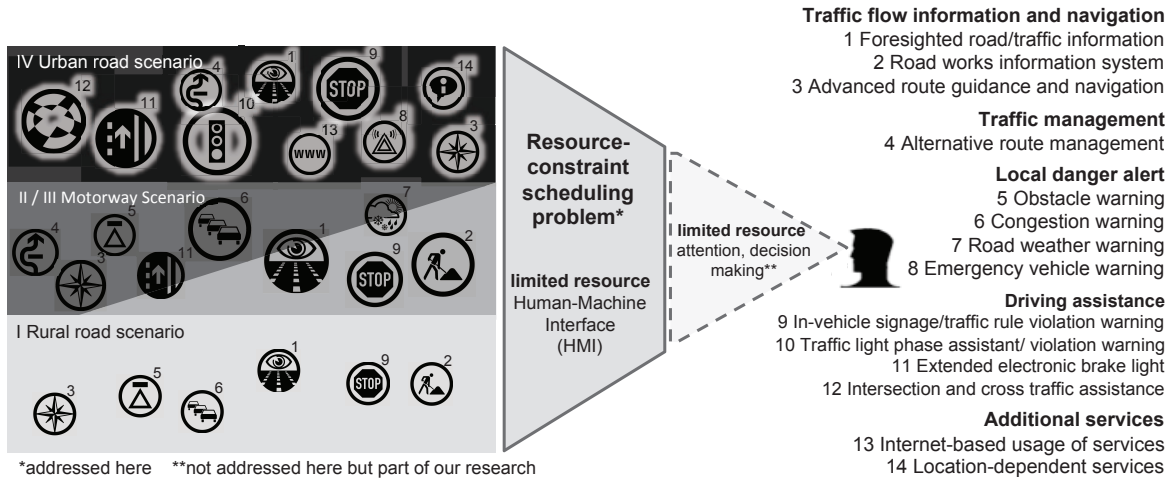


Figure 2: Complexity of the scenarios I–IV and the role of RCSP in the given application context.

**Algorithm 1:** Scheduling using BFS tree search.

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1: solution ← nil
2: minPenalty ← ∞
3: i ← 0
4:  $\mathcal{N}_0 \leftarrow \{R\}$ 
5: while  $|\mathcal{N}_i| > 0$  do
6:   for all  $n \in \mathcal{N}_i$  do
7:     for all  $m \in \mathcal{M}$  do
8:       if applicable(m,n) then
9:          $n' \leftarrow \text{apply}(m,n)$ 
10:         $\mathcal{N}_{i+1} \leftarrow \mathcal{N}_{i+1} \cup \{n'\}$ 
11:        if isSolution( $n'$ ) and  $p(n') < \text{minPenalty}$  then
12:          minPenalty ←  $p(n')$ 
13:          solution ←  $n'$ 
14:        end if
15:      end if
16:    end for
17:  end for
18:   $i \leftarrow i+1$ 
19: end while

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**Pruning.** In order to reduce complexity and runtime, several pruning mechanisms are applied in the algorithm: A branch with a total penalty bigger than the best solution found so far minus the minimum action penalty will be pruned; a branch that encodes a solution already encoded in another branch (maybe with permuted actions) will be pruned. Additional implicit pruning is achieved using the function *applicable(a,n)* (see line 8 of Algorithm 1):

**Definition 4** (applicable(m,n)). A modification  $m$  is applicable to a node  $n$  if and only if:

- $n$  is not a leaf
- applying  $m$  to  $n$  will not create another conflict

- $m$  will not shorten any task below a given minimal display time
- $p(n') < \text{minPenalty}$
- $n' \notin \mathcal{N}_j, 0 \leq j \leq i+1$
- $m$  will not move any task completely out of its original scope

**Anytime Behavior.** Our algorithm shows an "anytime" behavior, i.e. it incrementally finds better plans (Baier et al., 2007). Once a solution is found, its metric value can be used as a bound for future solutions.

The metric we use here is a penalty of the actions needed in order to modify the conflict set to a conflict-free set. It is defined in more detail below.

## 5 EMPIRICAL EVALUATION

In preparation of the SIM<sup>TD</sup> field test which will begin in 2012, applications are tested on the basis of pre-recorded traces containing GPS positions, car-to-car communication messages, and vehicle data (velocity, heading, gas pedal status, break pedal status, index lights, etc.). The scenarios lined-up at the X-axis in Figure 3 correspond to what we have seen earlier in Figures 1 and 2. Additionally, a scenario with hyper-realistic complexity was added (here denoted Scenario V). Each scenario was further specified with three concrete evaluation cases (I.1, I.2, I.3, .... V.3). The dashed black line corresponds to the actual complexity of the evaluation cases:

**Definition 5** (Complexity). The complexity of a conflict set  $T^C$  is

$$cx(T^C) = \frac{\sum ol(t_i^C, t_j^C)}{ms} \cdot |T^C| \cdot |ol(t_i^C, t_j^C)| \cdot alloc(ms),$$

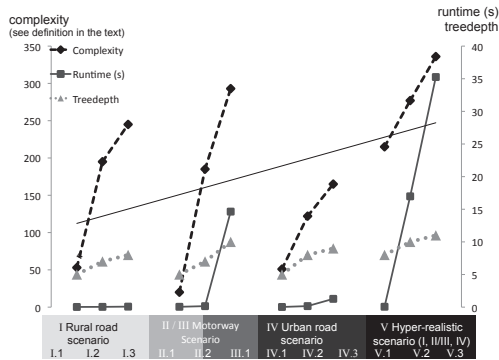


Figure 3: For each type of scenario, several examples of varying complexity based on field data are used to test the algorithm presented in this paper. Hyperrealistic in the sense of unrealistic high) scenarios are used for stress testing. Runtime and tree depth match the tendency of growing complexity from left to right in each scenario type.

where  $ol$  is the temporal overlap of two conflicting tasks in milliseconds and  $alloc(ms)$  is the percentage of the makespan allocated by the unmodified tasks.

Figure 3 shows that the complexity of the test cases has a positive trend from left to right (see solid black trend line), which appeals to our intuition. Nevertheless, there is a significant case-specific complexity within each of the scenarios. We see, for example, that the variance of complexity in *Motorway scenario* is higher than both *Rural road scenario* and *Urban road scenario*. Note that the complexity of V.1 is lower than the ones of I.3 and III.1. This is because V.1 is a hyper-real *Rural road* test case, V.2 is hyper-real *Motorway* test case, and V.3 is a hyper-real *Urban road* test case.

Both run-time as well as tree-depth differed significantly, which can be seen by regarding the grey (square) solid lines respectively the dashed (triangle) lines. Runtime relates to the time needed to find the optimal solution. It was measured on a regular state-of-the-art desktop PC.

For each of the evaluation cases, a solution was found by the scheduling algorithm. In order to be able to distinguish between successful solutions of different quality, we define quality via a penalty for the modifying actions it involves: the fewer the modifications, the better the solution.

**Definition 6** (Penalty of a RCSP Solution). *Be  $\mathcal{M}$  a set of modifications,  $\mathcal{M} = \{m_1, \dots, m_N\}$  and  $t$  a task with given positive priority  $prio(t)$ . The penalty  $p$  for  $m$  applied to  $t$  is  $p(m, t) = d(m) \cdot prio(t) \cdot \Delta$ , where  $d(m)$  defines the general desirability of an action as positive interger value and  $\Delta$  specifying the temporal aspect of the modification (e.g. postponing or shortening).*

The penalty of a solution is then defined as

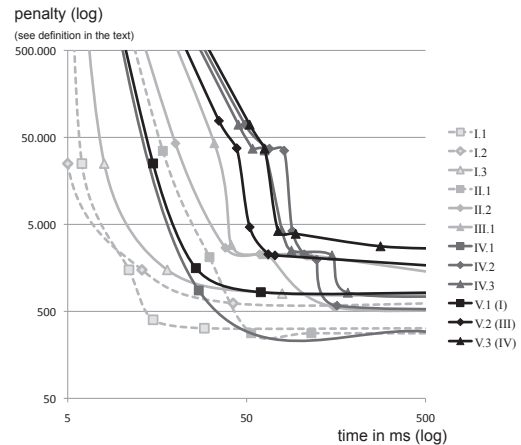


Figure 4: Quality of a solution annotated as penalty (see Definition 6) as a function of runtime (ms). Solutions with qualities close to the respective optimal solutions are found after a very short time. Note that presentation tasks are issued at least a few seconds before start time. Hence, the planning time does not delay the warning.

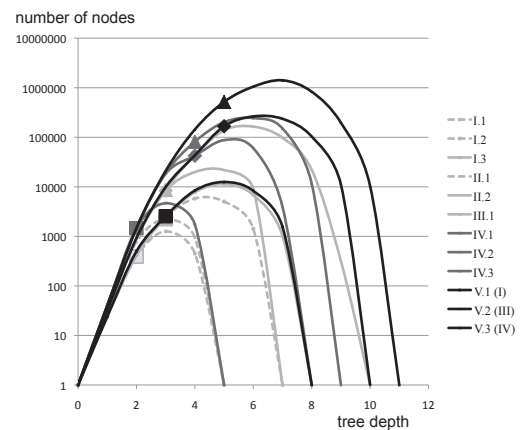


Figure 5: Number of nodes as a function of tree depth showing the effect of pruning. The symbols correspond to the point when the optimal solution was found.

$\sum p(m, t)$  of all  $(m, t)$  contained in it.

The order of steps taken towards a certain solution is in our case not important. This is respected by pruning duplicate and redundant solutions. As a side effect, the penalty for a solution is well defined and independent of the way it was constructed.

Figure 4 shows the correlation between runtime and solution quality in terms of the penalty we defined. Our algorithm provides a first solution typically in the first few milliseconds and then finds increasingly better solutions fast. After 50-100 ms, we are very close to, or in some cases already at, the optimal solution. Note that the optimal solution is not at penalty zero, so the asymptotic behaviour of the graph is not aimed towards the X-axis. Interrupting

the scheduling process at this point leads to a satisfactory result.

In Figure 5, the effects of pruning are depicted. The tree depth is correlated to the number of nodes calculated on that level using our BFS tree search. If no pruning was applied, the graph would be a straight line on the logarithmic scale. Using the penalty of the best solution found so far (which drops rather rapidly as we saw in Figure 4) as an upper bound enables us to severely prune the tree fast. The symbols correspond to the point when the optimal solution was found.

## 6 CONCLUSIONS AND OUTLOOK

In this paper, we presented the Resource-constrained Scheduling Problem (RCSP), which is like the well known Resource-constrained Project Scheduling Problem (RCPS) a combinatorial optimization problem. The automotive domain, in which we encountered this problem, is highly dynamic and requires fast responses, e.g. an anytime behaviour of the algorithm tackling this problem. We presented a tree-search based solution for the RCSP, analyzed its runtime behaviour, solution quality increase and space consumption. Although the algorithm is space-consuming when running until termination, very good results are produced fast and the calculation can be stopped then. We argue that the solution presented is suitable for the scenario at hand. As a next step, we will alternatively implement a genetic algorithm for the problem, analyze it and compare it to the performance of the algorithm presented here. This will, based on a categorization of problems according to its complexity, lead to a hybrid approach using the most suitable mechanism for each problem.

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